

SONINE INTEGRAL EQUATIONS OF THE FIRST KIND IN $L_p(0, b)$

Stefan G. Samko ¹ and Rogério P. Cardoso ²

Abstract

A Volterra integral equation of the first kind

$$\mathbf{K}\varphi(x) := \int_0^x k(x-t)\varphi(t) dt = f(x), \quad 0 < x < b \leq \infty,$$

with a kernel $k(x) \in L_1(0, b)$ is called *Sonine equation*, if there exists another locally integrable kernel $\ell(x)$ such that $\int_0^x k(x-t)\ell(t) dt \equiv 1$. We construct the real inverse operator, within the framework of the spaces $L_p(0, b)$, in Marchaud form:

$$\mathbf{K}^{-1}f(x) = \ell(x)f(x) + \int_0^x \ell'(t)[f(x-t) - f(x)] dt$$

with the interpretation of the convergence of this "hypersingular" integral in L_p -norm. The description of the range $K(L_p)$ is given.

Mathematics Subject Classification: 45D05, 45H05, 44A15, 26A33

Key Words and Phrases: Sonine kernels, integral equations of the first kind, special functions, Marchaud constructions, Hardy inequality